CCGPS Frameworks Student Edition

Mathematics

Fifth Grade Unit Seven
Volume and Measurement

Dr. John D. Barge, State School Superintendent
"Making Education Work for All Georgians"

Unit 7
VOLUME AND MEASUREMENT

TABLE OF CONTENTS

OVERVIEW

CONVERT LIKE MEASUREMENT UNITS WITHIN A GIVEN MEASUREMENT SYSTEM.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ***conversion/convert, metric and customary measurement From previous grades: relative size, liquid volume, mass, length, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), hour, minute, second***

GEOMETRIC MEASUREMENT: UNDERSTAND CONCEPTS OF VOLUME AND RELATE VOLUME TO MULTIPLICATION AND TO ADDITION.

Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ***measurement, attribute, volume, solid figure, right rectangular prism, unit, unit cube, gap, overlap, cubic units (cubic cm, cubic in. cubic ft. nonstandard cubic units), multiplication, addition, edge lengths, height, area of base.***

REPRESENT AND INTERPRET DATA

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ***line plot, length, mass, liquid volume.***

In this unit students will:

- recognize volume as an attribute of three-dimensional space.
- understand that volume can be measured by finding the total number of same size units of volume required to fill the space without gaps or overlaps.
- understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume.
- select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume.
- decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes.

- measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.
- communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language.

Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MCC5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MCC5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

MCC5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

MCC5. MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were*

COMMON MISCONCEPTIONS

When solving problems that require regrouping units, students use their knowledge of regrouping the numbers as with whole numbers. Students need to pay attention to the unit of measurement which dictates the regrouping and the number to use. The same procedures used in regrouping whole numbers should not be taught when solving problems involving measurement conversions.

STANDARDS FOR MATHEMATICAL PRACTICE

This section provides examples of learning experiences for this unit that support the development of the proficiencies described in the Standards for Mathematical Practice. These proficiencies correspond to those developed through the Literacy Standards. The statements provided offer a few examples of connections between the Standards for Mathematical Practice and the Content Standards of this unit. The list is not exhaustive and will hopefully prompt further reflection and discussion.

1. **Make sense of problems and persevere in solving them.** Students make sense that the square units represent 2-dimensional objects and have both length and width.

2. **Reason abstractly and quantitatively.** Students demonstrate abstract reasoning to create a display of square and cubic units in order to compare/contrast the measures of area and volume.

3. **Construct viable arguments and critique the reasoning of others.** Students construct and critique arguments regarding their knowledge of what they know about area and volume.

4. **Model with mathematics.** Students use snap cubes to build cubes and rectangular prisms in order to generalize a formula for the volume of rectangular prisms.

5. **Use appropriate tools strategically.** Students select and use tools such as tables, cubes, and other manipulatives to represent situations involving the relationship between volume and area.

6. **Attend to precision.** Students attend to the precision when comparing and contrasting the prisms made using the same amount of cubes.

7. **Look for and make use of structure.** Students recognize volume as an attribute of solid figures and understand concepts of volume measurement. Students use their understanding of the mathematical structure of area and apply that knowledge to volume.

8. **Look for and express regularity in repeated reasoning.** Students relate new experiences to experiences with similar contexts when studying a solid figure that can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson*****

ENDURING UNDERSTANDINGS

- Three-dimensional (3-D) figures are described by their faces (surfaces), edges, and vertices (singular is “vertex”).
- Volume can be expressed in both customary and metric units.
- Volume is represented in cubic units – cubic inches, cubic centimeters, cubic feet, etc.
- Volume refers to the space taken up by an object itself.

BIG IDEAS *From Teaching Student Centered Mathematics, Van de Walle & Lovin, 2006.*

- Volume is a term for measures of the “size” of three-dimensional regions.
- Volume typically refers to the amount of space that an object takes up.
- Volume is measured with units such as cubic inches or cubic centimeters-units that are based on linear measures.
- Two types of units can be used to measure volume: solid units and containers.

ESSENTIAL QUESTIONS

- How do we measure volume?
- How are area and volume alike and different?
- How can you find the volume of cubes and rectangular prisms?
- How do you convert volume between units of measure?
- What is the relationship between the volumes of geometric solids?
- Why are some tools better to use than others when measuring volume?
- Why is volume represented with cubic units and area represented with square units?

CONCEPTS/SKILLS TO MAINTAIN

MCC.5.MD.3, MCC.5.MD.4, and **MCC.5.MD.5:** *These standards represent the first time that students begin exploring the concept of volume. In third grade, students begin working with area and covering spaces. The concept of volume should be extended from area with the idea that students are covering an area (the bottom of cube) with a layer of unit cubes and then adding* *layers of unit cubes on top of bottom layer (see picture below). Students should have ample experiences with concrete manipulatives before moving to pictorial representations. Students' prior experiences with volume were restricted to liquid volume. As students develop their understanding volume they understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit. This cubic unit is written with an exponent of 3 (e.g., in^3, m^3). Students connect this notation to their understanding of powers of 10 in our place value system. Models of cubic inches, centimeters, cubic feet, etc are helpful in developing an image of a cubic unit. Students estimate how many cubic yards would be needed to fill the classroom or how many cubic centimeters would be needed to fill a pencil box.*

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- number sense
- computation with whole numbers and decimals, including application of order of operations
- addition and subtraction of common fractions with like denominators
- angle measurement
- measuring length and finding perimeter and area of rectangles and squares
- characteristics of 2-D and 3-D shapes
- data usage and representations
- convert metric and customary units within units of measure

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The terms below are for teacher reference only and are not to be memorized by the students.

- measurement
- attribute
- volume
- solid figure
- right rectangular prism
- unit
- unit cube
- gap

- overlap
- cubic units (cubic cm, cubic in, cubic ft, nonstandard cubic units
- edge lengths
- height
- area of base

COMMON CORE GLOSSARY

http://www.corestandards.org/Math/Content/mathematics-glossary/glossary

STRATEGIES FOR TEACHING AND LEARNING

Convert like measurement units within a given measurement system.
MCC.5.MD.1

This standard calls for students to convert measurements within the same system of measurement in the context of multi-step, real-world problems. Both customary and standard measurement systems are included; students worked with both metric and customary units of length in second grade. In third grade, students work with metric units of mass and liquid volume. In fourth grade, students work with both systems and begin conversions within systems in length, mass and volume.

To convert from one unit to another unit in the standard and metric system, the relationship between the units must be known. In order for students to have a better understanding of the relationships between units, they need to use measuring tools in class. The number of units must relate to the size of the unit.
Example 1: 100 cm = 1 meter
Example 2: 12 inches = 1 foot and 3 feet = 1 yard

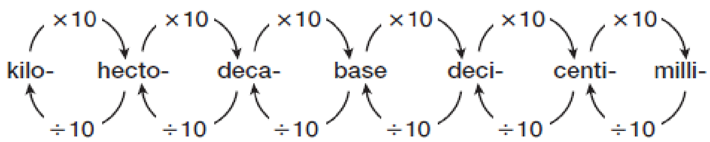
When converting in the metric system, have students extend their prior knowledge of the base-ten system as they multiply or divide by powers of ten (as referenced in Units 1 and 2). Teaching conversions should focus on the relationship of the measurements, not merely rote memorization. The questions ask the student to find out the size of each of the subsets. Students are not expected to know e.g. that there are 5280 feet in a mile. If this is to be used as an assessment task, the conversion factors should be given to the students. However, in a teaching situation it is worth having them realize that they need that information rather than giving it to them upfront; having students identify what information they need to have to solve the problem and knowing where to go to find it allows them to engage in Standard for Mathematical Practice 5, Use appropriate tools strategically.

Retrieved from Illustrative Mathematics
http://www.illustrativemathematics.org/standards/k8

The metric system of measurement is based on 10 and powers of 10. The prefixes used for length, capacity, and mass tell what part of the basic unit is being considered. The symbols for each unit of measure are given in parentheses ( ). The most commonly used units are shown in **bold** below.

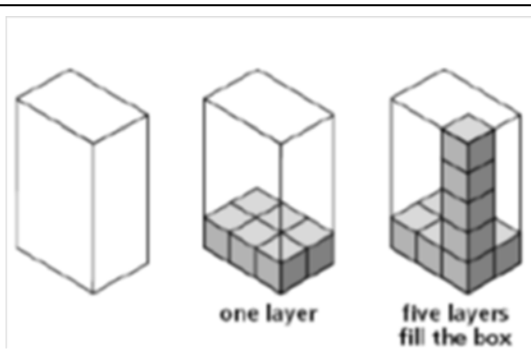
prefix	meaning	measure of length	measure of capacity	measure of mass
kilo-	1,000	**kilometer** (km)	kiloliter (kL)	**kilogram** (kg)
hecto-	100	hectometer (hm)	hectoliter (hL)	hectogram (hg)
deka-	10	dekameter (dkm)	dekaliter (dkL)	dekameter (dkg)
base unit	1	**meter** (m)	**liter** (L)	**gram** (g)
deci-	0.1	**decimeter** (dm)	deciliter (dL)	decigram (dg)
centi-	0.01	**centimeter** (cm)	centiliter (cL)	centigram (cg)
milli-	0.001	**millimeter** (mm)	**milliliter** (mL)	**milligram** (mg)

To change from a larger unit to a smaller unit, multiply by the appropriate power of 10.
To change from a smaller unit to a larger unit, divide by the appropriate power of 10.



Geometric measurement: Understand Concepts of volume and relate volume to multiplication and to addition.
MCC.MD.3 – MCC.MD.4 – MCC.MD.5

These standards involve finding the volume of right rectangular prisms and extend their understanding of finding the area of composite figures into the context of volume. Students should have experiences to describe and reason about why the formula is true. Specifically, that they are covering the bottom of a right rectangular prism (length x width) with multiple layers (height). Therefore, the formula (length × width × height) is an extension of the formula for the area of a rectangle.

	(3 × 2) represents the number of blocks in the first layer (3 × 2) × 5 represents the number of blocks in 5 layers 6 × 5 represents the number of block to fill the figure 30 blocks fill the figure

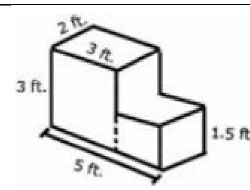
Example:

When given 24 cubes, students make as many rectangular prisms as possible with a volume of 24 cubic units. Students build the prisms and record possible dimensions.

Length	Width	Height
1	2	12
2	2	6
4	2	3
8	3	1

Example:

Students determine the volume of concrete needed to build the steps in the diagram at the right.

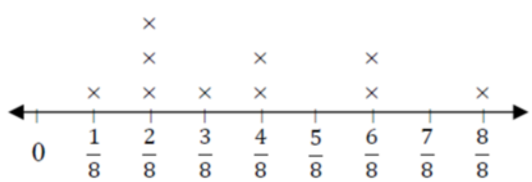


REPRESENT AND INTERPRET DATA

MCC5.MD.2

This standard provides a context for students to work with fractions by measuring objects to one-eighth of a unit. This includes length, mass, and liquid volume. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

<u>Example 1:</u>
Students measured objects in their desk to the nearest 1/2, 1/4, or 1/8 of an inch then displayed data collected on a line plot. How many objects measured 1/4? 1/2? If you put all the objects together end to end what would be the total length of all the objects?



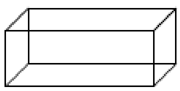
<u>Instructional Strategies (Volume and Measurement)</u>

Volume refers to the amount of space that an object takes up and is measured in cubic units such as cubic inches or cubic centimeters.

Students need to experience finding the volume of rectangular prisms by counting unit cubes, in metric and standard units of measure, before the formula is presented. Provide multiple opportunities for students to develop the formula for the volume of a rectangular prism with activities similar to the one described below.

Give students one block (a 1- or 2- cubic centimeter or cubic-inch cube), a ruler with the appropriate measure based on the type of cube, and a small rectangular box. Ask students to determine the number of cubes needed to fill the box. Have students share their strategies with the class using words, drawings or numbers. Allow them to confirm the volume of the box by filling the box with cubes of the same size.

By stacking geometric solids with cubic units in layers, students can begin understanding the concept of how addition plays a part in finding volume. This will lead to an understanding of the formula for the volume of a right rectangular prism, b x h, where b is the area of the base. A right rectangular prism has three pairs of parallel faces that are all rectangles.



Have students build a prism in layers. Then, have students determine the number of cubes in the bottom layer and share their strategies. Students should use multiplication based on their knowledge of arrays and its use in multiplying two whole numbers.

Instructional Resources/Tools

- Cubes
- Rulers (marked in standard or metric units)
- Grid paper

http://illuminations.nctm.org/ActivityDetail.aspx?ID=6: Determining the Volume of a Box by Filling It with Cubes, Rows of Cubes, or Layers of Cubes
This cluster is connected to the third Critical Area of Focus for Grade 5, **Developing understanding of volume.**

EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- Identify faces, edges, and vertices of cubes and rectangular prisms.
- Understand volume can be determined by finding the product of the area of the base times the height $V = B \times h$. or $V=l\ x\ w\ x\ h$
- Estimate and determine the volume of cubes and rectangular prisms.
- Compare the volume of different objects with and without formulae.
- Convert volume measurements within a single system of measurement (customary, metric).
- Measure solid cubes and rectangular prisms using standard customary and metric measures

TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all fourth grade students. These tasks or tasks of similar depth and rigor should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they also may be used for teaching and learning.

Scaffolding Task	Tasks that build up to the learning task.
Learning Task	Constructing understanding through deep/rich contextualized problem solving tasks.
Practice Task	Tasks that provide students opportunities to practice skills and concepts.
Performance Task	Tasks which may be a formative or summative assessment that checks for student understanding/misunderstanding and or progress toward the standard/learning goals at different points during a unit of instruction.
Culminating Task	Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning.
Formative Assessment Lesson (FAL)	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards.
CTE Classroom Tasks	Designed to demonstrate how the Common Core and Career and Technical Education knowledge and skills can be integrated. The tasks provide teachers with realistic applications that combine mathematics and CTE content.

Task Name	**Task Type** ***Grouping Strategy***	**Content Addressed**	**Standard(s)**
Differentiating Area and Volume	Scaffolding Task *Small Group Task*	Investigate the relationships between area and volume	MCC5.MD.3a-b
How Many Ways?	Constructing Task *Individual/Partner Task*	Develop a formula for determining the volume of cubes and rectangular prisms	MCC5.MD.3a-b MCC5.MD.4 MCC5.MD.5a-c MCC5.MD.1
Exploring with Boxes	Practice Task *Individual/Partner Task*	Use a chart to find volume	MCC5.MD.3a-b MCC5.MD.4 MCC5.MD.5a-c
Rolling Rectangular Prisms	Practice Task *Individual/Partner Task*	Find the volume of rectangular prisms	MCC5.MD.1 MCC5.MD.3a-b MCC5.MD.5a-c

Books, Books, and More Books	Constructing Task *Individual/Partner*	Add to find the combined volume of multiple rectangular prisms	MCC5.MD.3a-b MCC5.MD.4 MCC5.MD.5a-c
Super Solids	Practice Task *Individual/Partner Task*	Estimate and calculate the volume of rectangular prisms	MCC5.MD.5a-c
Survival Badge	Practice Task *Individual/Partner Task*	Use line plots to display a set of measurements	MCC5.MD.2
A Little Mo Running	Practice Task *Individual/Partner Task*	Use line plots to display a set of measurements	MCC5.MD.2
Toy Box Designs	Performance Task *Individual/Partner Task*	Design a toy box with a given volume	MCC5.MD.3a-b MCC5.MD.4 MCC5.MD.5a-c
Breakfast for All	Performance Task *Individual/Partner Task*	Create 3 different sized boxes for cereal	MCC5.MD.3a-b MCC5.MD.4 MCC5.MD.5a-c
Boxing Boxes	**Culminating Task** *Individual/Partner Task*	Consider volume and capacity to determine guidelines for packing boxes	MCC5.MD.1 MCC5.MD.3a-b MCC5.MD.4 MCC5.MD.5a-c

If you need further information about this unit visit the GaDOE website and reference the unit webinars.
https://www.georgiastandards.org/Common-Core/Pages/Math-PL-Sessions.aspx

SCAFFOLDING TASK: Differentiating Area and Volume

Students create a display of square and cubic units in order to compare/contrast the measures of area and volume.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

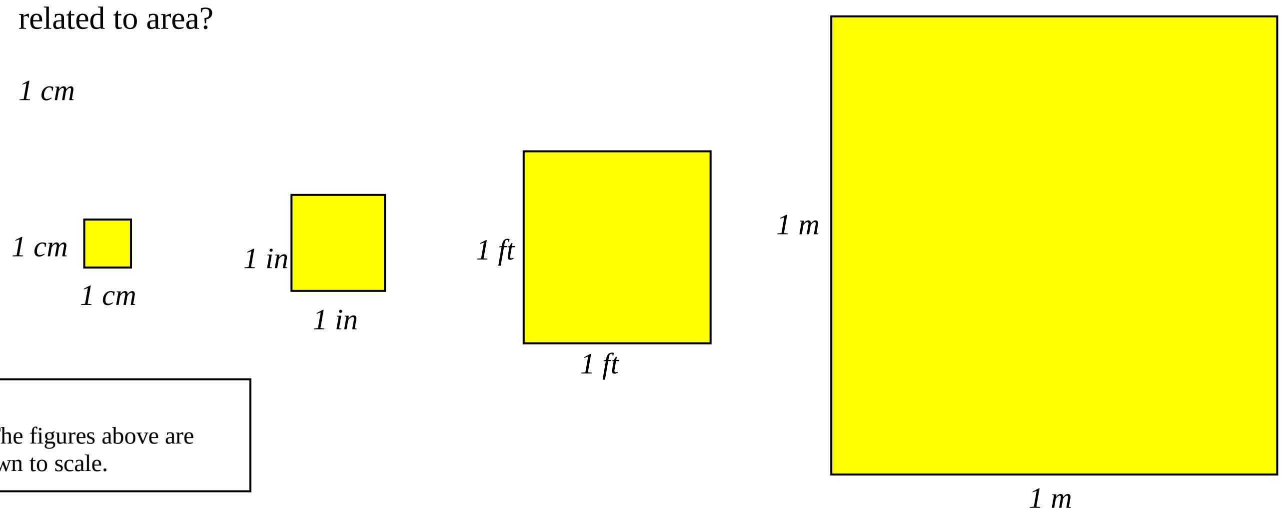
b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

STANDARDS FOR MATHEMATICAL PRACTICE

SMP 1. Make sense of problems and persevere in solving them.
SMP 2. Reason abstractly and quantitatively.
SMP 3. Construct viable arguments and critique the reasoning of others.
SMP 4. Model with mathematics.
SMP 5. Use appropriate tools strategically.
SMP 6. Attend to precision.
SMP 7. Look for and make use of structure.
SMP 8. Look for and express regularity in repeated reasoning.

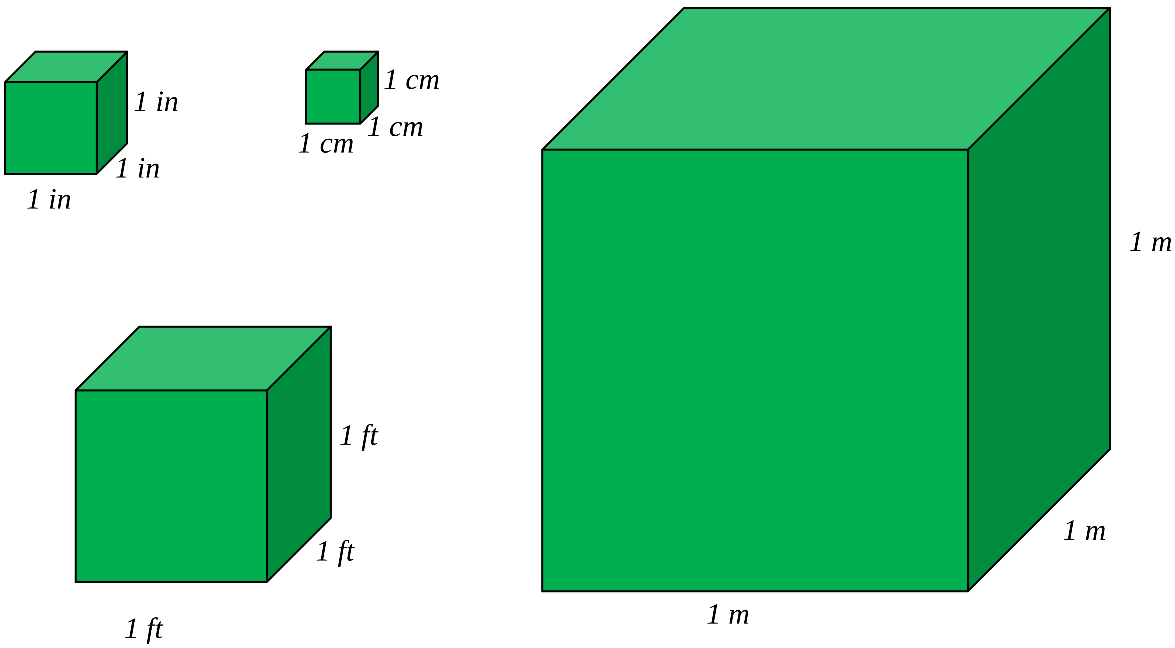
BACKGROUND KNOWLEDGE

Students should realize that the square units represent 2-dimensional objects and have both length and width. If students are having difficulty determining how to create these, have a class discussion about the word "square." What comes to mind? How do you think this word might be related to area?



Note: The figures above are not drawn to scale.

Students should also realize that the cubic units represent 3-dimensional objects and have length, width, and height. If students are having difficulty determining how to create these, have a class discussion about the words "cube" and "cubic." What comes to mind? How do you think these words might be related to volume?



Note: The figures above are not drawn to scale.

<u>Common Misconceptions:</u>
Some students may think the term "square" refers only to the geometric figure with equal length sides. They will need to understand that area of any rectangle is measured in square units. The same idea may be present in "cubic units". Students may think it only has to do with the geometric solid "cube". They need to understand that "cubic units" are used to measure any rectangular prism.

ESSENTIAL QUESTIONS

- Why is volume represented with cubic units and area represented with square units?
- How are area and volume alike and different?

MATERIALS

- "Differentiating Area and Volume" student recording sheet
- newspaper
- construction paper
- copy paper
- grid paper (cm, in)

- scissors
- masking tape
- rulers
- meter sticks
- measuring tape
- cardstock or poster board
- markers

GROUPING

Small Group

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Students create a display of square and cubic units in order to compare/contrast the measures of area and volume.

Comments

This is a cooperative learning activity in problem solving. Students are provided with materials, but no initial instruction is given on how to build the models. This task will help give students a tangible model of square units and cubic units.

To open this task, students can discuss in their small groups what they know about area and volume. Key points of a class discussion can be recorded on chart paper.

Students will work in small groups to build models to represent units of area and units of volume. When the groups have completed their projects they will share with the class what they built, what each is called, and how each compares to some of the other models built by other groups.

Task Directions

Students will follow the directions below from the “Differentiating Area and Volume” student recording sheet.

Create a display for area and volume by creating the following models. Use newspaper, construction paper, copy paper, grid paper, scissors, masking tape, meter sticks, markers and/or cardboard to build the models.

- Area models – 1 cm^2, 4 cm^2, 1 in^2, 4 in^2, 1 ft^2
- Volume models – 1 cm^3, 8 cm^3, 1 in^3, 8 in^3, 1 ft^3

At the end of the work period, each group will share their completed models and explain what has been built, what each is called, and how your models compare with some of the other models built by the other groups.

Individually, answer the following questions:

- How are area and volume alike?

- How are area and volume different?
- Why is area labeled with square units?
- Why is volume labeled with cubic units?
- Think about your home – bedroom, kitchen, bathroom, living room.
  - What would you measure in square units? Why?
  - What would you measure in cubic units? Why?

FORMATIVE ASSESSMENT QUESTIONS

- What does cm^2 mean? cm^3? How do you know?
- What does in^2 mean? in^3? How do you know?
- What does ft^2 mean? ft^3? How do you know?
- What objects in everyday life could you use to represent cm^2? cm^3? in^2? in^3? ft^2? ft^3?
- How can you create a shape that represents 4 cm^2? What length would you use? How do you know?
- How can you create a shape that represents 8 cm^3? What length would you use? How do you know?

DIFFERENTIATION

Extension

- Ask students to describe the relationship between 4 cm^2 and 8 cm^3 as well as 9 cm^2 and 27 cm^3. Then have students generate other pairs of numbers that have the same relationship. What do they notice? (Students may use 1 cm cubes placed on a 4 cm^2 or 9 cm^2 square to determine the dimensions of a cube built on the square.)

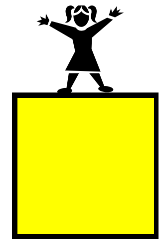
Intervention

- Allow students to create at least some of the figures using a word processing or a drawing computer program. This will allow students to easily create right angles, equal side lengths, and cubes with equal edge lengths.
- Students may benefit from using 1” square tiles, 1” cubes, and similar 1 cm materials to create some of these models, especially 4 cm^2, 4 in^2, 8 cm^3, and 8 in^3**.**

TECHNOLOGY

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear

dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.



Name___________________________Date _____________________

Differentiating Area and Volume

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At the end of the work period, each group will share their completed models and explain what has been built, what each is called, and how your models compare with some of the other models built by the other groups.

Individually, answer the following questions:

1. How are area and volume alike?

2. How are area and volume different?

3. Why is area labeled with square units?

4. Why is volume labeled with cubic units?

____

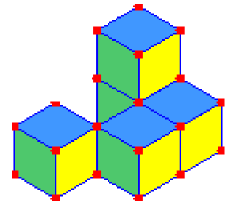
5. Think about your home – bedroom, kitchen, bathroom, living room.

   What would you measure in square units? Why?

   ____

   What would you measure in cubic units? Why?

   ____



CONSTRUCTING TASK: How Many Ways?

In this task, students will use 24 snap cubes to build cubes and rectangular prisms in order to generalize a formula for the volume of rectangular prisms.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

MCC5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MCC5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MCC5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

SMP 1. Make sense of problems and persevere in solving them.
SMP 2. Reason abstractly and quantitatively.
SMP 3. Construct viable arguments and critique the reasoning of others.
SMP 4. Model with mathematics.
SMP 5. Use appropriate tools strategically.
SMP 6. Attend to precision.

SMP 7. Look for and make use of structure.
SMP 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students should have had experiences with the attributes of rectangular prisms, such as faces, edges, and vertices, in fourth grade. This task will build upon this understanding.

The "How Many Ways?" student recording sheet asks students to determine the area of the base of each prism using the measurements of base and height of the solid's BASE. The general formula for the area of a parallelogram is $A = bh$. Knowing the general formula for the area of a parallelogram enables students to memorize ONE formula for the area of rectangles, squares, and parallelograms since each of these shapes is a parallelogram.

The general formula for the volume of a prism is $V = Bh$, where B is the area of the BASE of the prism and h is the height of the prism. Knowing the general formula for the volume of a prism prevents students from having to memorize different formulas for each of the types of prisms they encounter.

There are six possible rectangular prisms that can be made from 24 snap cubes.

$1 \times 1 \times 24$
$1 \times 2 \times 12$
$1 \times 3 \times 8$
$1 \times 4 \times 6$
$2 \times 2 \times 6$
$2 \times 3 \times 4$

Students may identify rectangular prisms with the same dimensions in a different order, for example, $1 \times 4 \times 6$, $1 \times 6 \times 4$, $6 \times 1 \times 4$, $6 \times 4 \times 1$, $4 \times 1 \times 6$, $4 \times 6 \times 1$. All of these are the same rectangular prism, just oriented differently. It is okay for students to include these different orientations on their recording sheet. However, some students may need to be encouraged to find different rectangular prisms.

Common Misconceptions:

Students may have difficulty with the concept of the formula $V=Bh$ representing 3 factors. (length, width, height). They may leave out one of the components because of that misconception.

ESSENTIAL QUESTIONS

- Why is volume represented with cubic units?
- How do we measure volume?
- How can you find the volume of cubes and rectangular prisms?
- In the formula $V=Bh$ *or,* $V=bh$, what does the B/b represent?

MATERIALS

- "How Many Ways?" student recording sheet

- Snap cubes

GROUPING
Partner/Small Group Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION
In this task, students will use 24 snap cubes to build cubes and rectangular prisms in order to generalize a formula for the volume of rectangular prisms.

Comments
To introduce this task ask students to make a cube and a rectangular prism using snap cubes. Discuss the attributes of cubes and rectangular prisms – faces, edges, and vertices. Initiate a conversation about the figures:

- What is the shape of the cube's base?
- What is the shape of the rectangular prism's base? The base of each is a rectangle (remember a square is a rectangle!).

Students should notice that the cube and rectangular prism are made up of repeated layers of the base. Describe the base of the figure as the first floor of a rectangular-prism-shaped building. Ask students, "What is the area of the base? Next, discuss the height of the figure. Ask students, "How many layers high is the cube?" or "How many layers high is the prism?" The number of layers will represent the height. DO NOT LEAD THE DISCUSSION TO THE VOLUME FORMULA. Students will use the results of this task to determine the volume formula for rectangular prisms on their own.

While working on the task, students do not need to fill in all ten rows of the "How Many Ways?" student recording sheet. Some students may recognize that there are only six different ways to create a rectangular prism using 24 snap cubes. For students who have found four or five ways to build a rectangular prism, tell them they have not found all of the possible ways **without telling them exactly how many ways are possible**. It is important for students to recognize when they have found all possible ways and to prove that they have found all of the possible rectangular prisms.

Once students have completed the task, lead a class discussion about the similarities and differences between the rectangular prisms they created using 24 snap cubes. Allow students to explain what they think about finding the volume of each prism they created. Also, allow students to share their conjectures about an efficient method to find the volume of any rectangular prism. Finally, as a class, come to a consensus regarding an efficient method for finding the volume of a rectangular prism.

Task Directions
Students will follow the directions below from the "How Many Ways?" student recording sheet.

1. Count out 24 cubes.

2. Build all the rectangular prisms that can be made with the 24 cubes. For each rectangular prism, record the dimensions and volume in the table below.
3. What do you notice about the rectangular prisms you created?
4. How can you find the volume without building and counting the cubes?

Shape #	Area of the BASE of the Solid A = *bh*		Number of Layers of the Base (Height of Solid)	Volume in cubic centimeters	Volume in cubic meters
	base	height			
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

FORMATIVE ASSESSMENT QUESTIONS

- What is the shape of the rectangular prism's base? Explain how to calculate the base of 3-dimensional objects.
- How did you determine the height of the rectangular prism? How do you know? (How many layers or "floors" does it have?)
- What is the volume of the rectangular prism? How do you know? (How many snap cubes did you use to make the rectangular prism? How do you know?)

DIFFERENTIATION

Extension

- Ask students to suggest possible dimensions for a rectangular prism that has a volume of 42 cm^3 without using snap cubes.

- Ask students to explore the similarities and differences of a rectangular prism with dimensions 3 cm x 4 cm x 5 cm and a rectangular prism with dimensions 5 cm x 3 cm x 4 cm. Students can consider the attributes and volumes of each of the prisms.
- Students can calculate the area of each surface of the solid and determine the total surface area.

Intervention

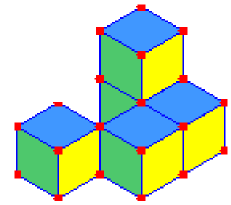
- Some students may need organizational support from a peer or by working in a small group with an adult. This person may help students recognize duplications in their table as well as help them recognize patterns that become evident in the table.
- Some students may benefit from using the "Cubes" applet on the Illuminations web site (see link in "Technology Connection" below). It allows students to easily manipulate the size of the rectangular prism and then build the rectangular prism using unit cubes.

TECHNOLOGY

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

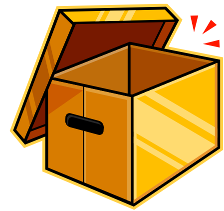
Name_______________________________________ Date ______________

How Many Ways?



1. Count out 24 cubes.
2. Build all the rectangular prisms that can be made with the 24 cubes. For each rectangular prism, record the dimensions and volume in the table below.
3. What do you notice about the rectangular prisms you created?
4. How can you find the volume without building and counting the cubes?

Shape #	Area of the BASE of the Solid A = *bh*		Number of Layers of the Base (Height of Solid)	Volume in cubic centimeters	Volume in cubic meters
	base	height			
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					



PRACTICE TASK: Exploring with Boxes

Adapted from K-5 Math Teaching Resources

In this task, students will create boxes and discover how volume is related to the length, width, and height of cubes.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MCC5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MCC5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

SMP 1. Make sense of problems and persevere in solving them.
SMP 2. Reason abstractly and quantitatively.
SMP 3. Construct viable arguments and critique the reasoning of others.
SMP 4. Model with mathematics.
SMP 5. Use appropriate tools strategically.
SMP 6. Attend to precision.
SMP 7. Look for and make use of structure.
SMP 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students should have experience with drawing boxes on grid paper. They also need to understand how to cut and fold the nets to make boxes. Teachers may need to model and let students practice before the task.

COMMON MISCONCEPTIONS:

When filling a 3-D figure, students may think there can be gaps or overlaps with the cubes filling the object. Have students compare this to finding the capacity of a container. If you put an object in that container, you would displace space to be filled with liquid, and you would get an inaccurate measure of the capacity. Same goes for a solid figure- there can be no displacement (unfilled space) if you want an accurate measurement.

ESSENTIAL QUESTIONS

- What is the relationship between the size of the box and the number of cubes it will hold?
- How does the volume change as the dimensions of the box change?

MATERIALS

- cube nets
- scissors
- tape
- cm cubes
- ruler
- recording sheet

GROUPING

Individual/Partners

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will create boxes and discover how volume is related to the length, width, and height of cubes.

Comments: To introduce this task, show the cube net and ask this question? What could be done to this net so that the top of the cube will be open? Students should discern that the top square could be cut off. Tell students that they will be building open cubes of different sizes and filling them with cubes. Explain that they will need to measure the dimensions of each cube to complete the chart.

Once students have completed the task, lead a class discussion about the patterns they noticed. Allow students to explain their findings and any relationships they noticed. Also, allow students to share their conclusions about the relationships between volume and the dimensions of cubes. Finally, allow students to write about their findings in their math journals.

Task Directions: Using the cube net, have students construct cubes of different dimensions and fill them with cm cubes. Have them measure the dimensions and record them in the appropriate boxes on the recording sheet. Then they will count the number of cubes it took to fill the cube and record the volume of each cube. Have students discuss their findings to generalize statements about the relationship between the dimensions of the cubes and their volume.

FORMATIVE ASSESSMENT QUESTIONS

- What do you notice about the size of the open cubes and the number of cm cubes they can hold? Explain your thinking.
- Could you predict how many cm cubes a container can hold, based on its measurements? Justify your answer.

DIFFERENTIATION

Extension:
- Students may create their own cubes using grid paper to create nets.
- Students may present a demonstration on drawing nets for cubes to the class.

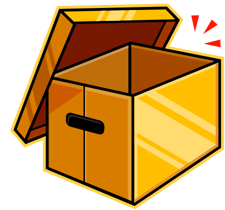
Intervention:
- Students may work with partners.
- Students may need support to measure dimensions accurately.
- Students may need support with differentiating between the length, width, and height on an open cube.

TECHNOLOGY

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

_________________________ (Name)　　　　　　　　　　____________________ (Date)

Exploring With Boxes



Materials: cube nets, scissors, tape, cm ruler, cm cubes, recording sheet

Directions:

1. Work with a partner. Cut out the nets for the open cubes, fold up the sides, and tape them together.
2. Measure each open cube and record your findings in the chart below.
3. Fill each box (open cube) with cm cubes and count them to find the volume.
4. Record your findings in the chart below.
5. Write in your math journal and describe how the size of the box is related to its volume.

Box (Open Cube)	Length of Base	Width of Base	Height of Cube	Volume
A				
B				
C				

Findings____

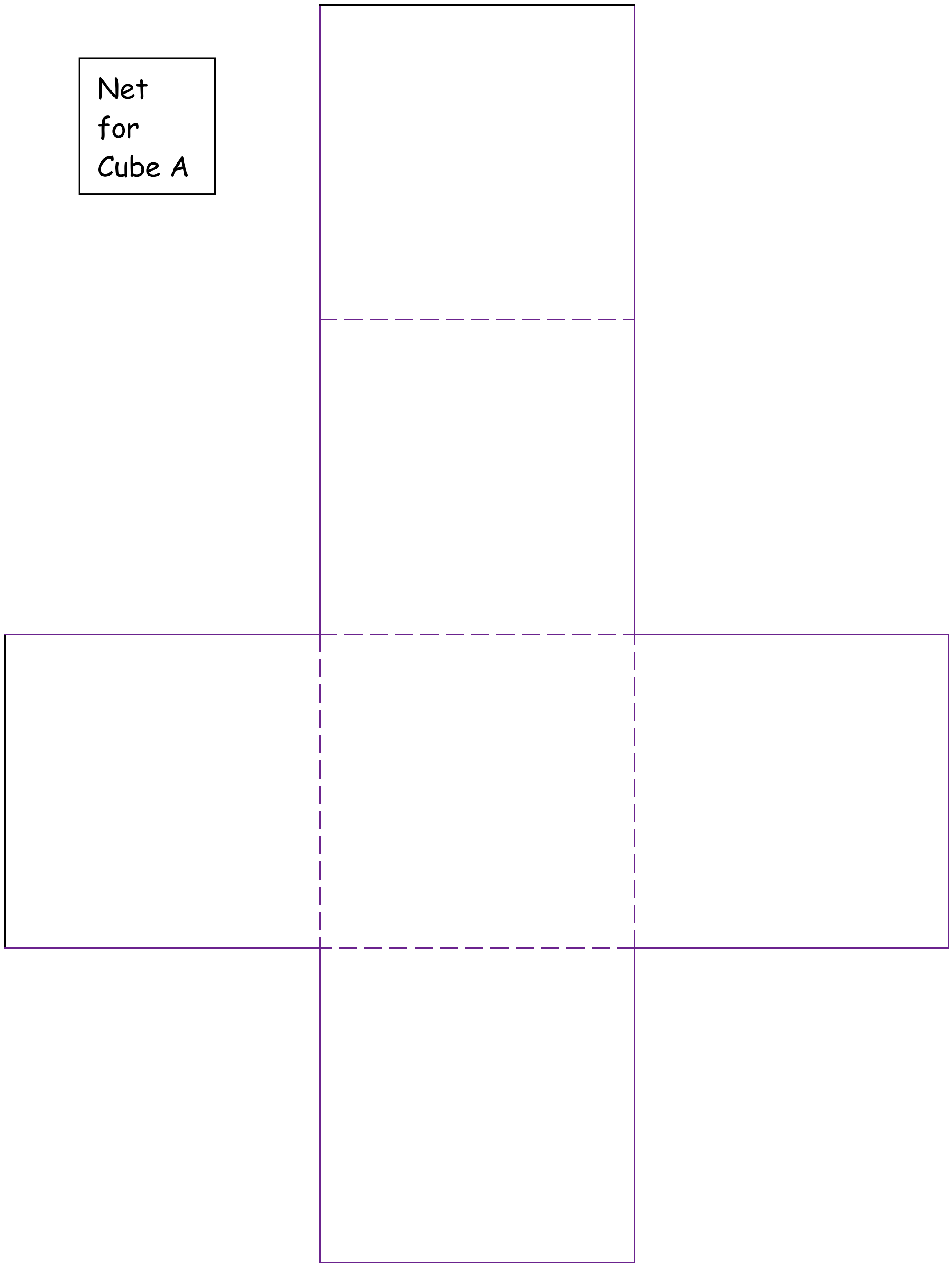
____

____

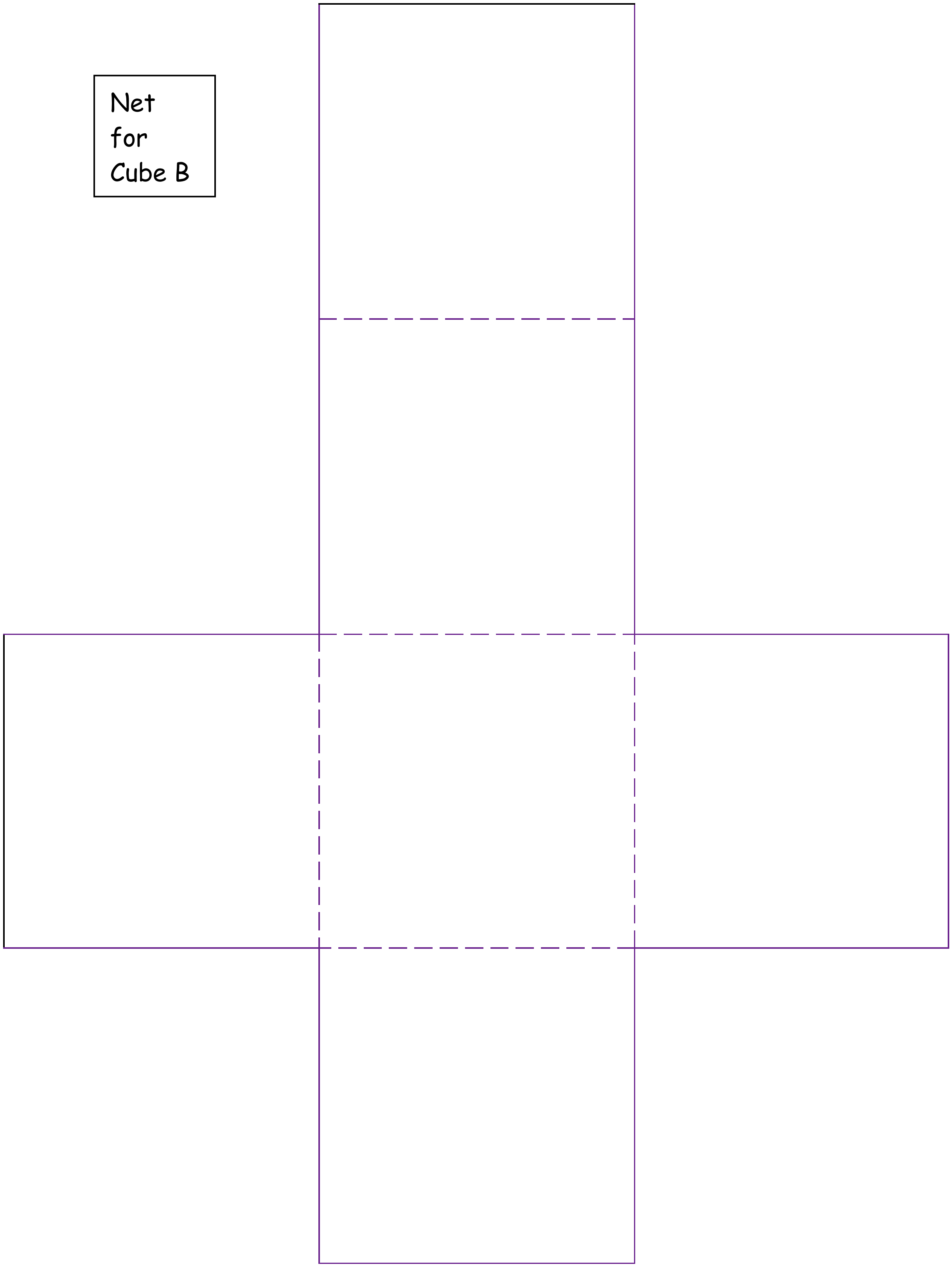
____

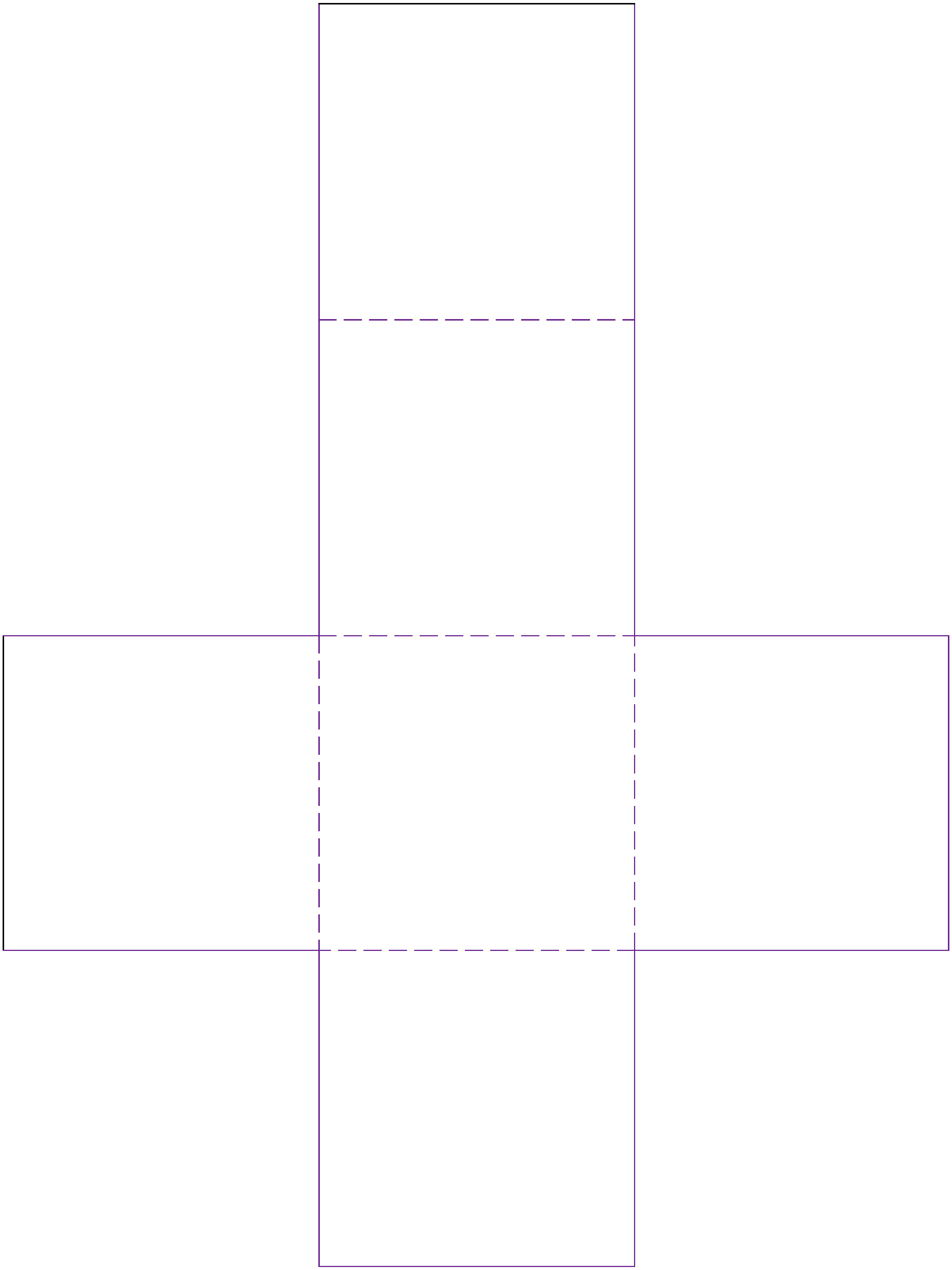
____

Net for Cube A



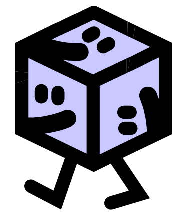
Net
for
Cube B





Net for

<u>PRACTICE TASK:</u> Rolling Rectangular Prisms



Adapted from K-5 Math Teaching Resources
In this task, students will draw and label rectangular prisms and roll a die to determine the measurements to calculate its volume.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

MCC5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MCC5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

MCC5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

SMP 1. Make sense of problems and persevere in solving them.
SMP 2. Reason abstractly and quantitatively.
SMP 3. Construct viable arguments and critique the reasoning of others.
SMP 4. Model with mathematics.
SMP 5. Use appropriate tools strategically.
SMP 6. Attend to precision.
SMP 7. Look for and make use of structure.

SMP 8. Look for and express regularity in repeated reasoning

BACKGROUND KNOWLEDGE

Students will need to know the names of the dimensions of rectangular prisms (length, width, height) and have some experience with the formulas $V = l \times w \times h$ and $V = b \times h$. Additionally, students will need to understand multiplication with 3 factors. They should also be familiar with converting metric units and customary units within systems.

COMMON MISCONCEPTIONS:

Students may believe that converting customary units is like converting metric units; using the base ten system. They will need to be reminded of equivalent measures in customary units if they are confused.

ESSENTIAL QUESTIONS

- Do all the dimensions have to be the same in a rectangular prism? Justify your answer.
- How are cubes and rectangular prisms the same? How are they different?
- How do you convert units from one measure to another in the metric system?
- How do you convert units from one measure to another in the standard/customary system?

MATERIALS

- Dice
- Recording sheet

GROUPING

- Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will draw and label rectangular prisms and roll a die to determine the measurements to calculate its volume.

Comments: To introduce this task, remind them of the formula for volume and that precision is very important in calculating volume.

Task Directions: Model drawing a rectangular prism and have someone roll the die to determine its measurements (length, width, and height) in inches. Label the drawing and model multiplying the three measurements to determine the volume. Have the students follow the directions on the task sheet to complete the task.
Part two of the task asks them to convert units within a system (metric or customary). They are accustomed to converting square units, so the conversion between cubic units of the same system should be easier. They will use the measurement of cubic inches and convert their answer into cubic feet.

FORMATIVE ASSESSMENT QUESTIONS

- What do you notice about the measurements and the volume of the rectangular prisms?
- What is the greatest possible volume for a rectangular prism in this game?
- What operation or steps do you use to convert measurements from a smaller to larger unit?
- What operation or steps do you use to convert measurements from a larger to smaller unit?

DIFFERENTIATION

Extension:

- Students may use both dice to increase the size of their rectangular prisms.
- Students may convert metric units of measure to millimeters.

Intervention:

- Students may work with partners.
- Students may use calculators to determine volume.

TECHNOLOGY

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

______________________________ ______________________________
Name **Date**

ROLLING A RECTANGULAR PRISM

Materials: dice, recording sheet

Directions:

1. **Draw a rectangular prism.**
2. **Roll a die three times to find the dimensions of the rectangular prism.**
3. **Label the dimensions.**
4. **Calculate the volume of the rectangular prism. Show your work.**
5. **Repeat steps 1-4 three times.**

Picture	Length	Width	Height	Volume in cubic inches	Volume in cubic feet

Name ____________________ ____ **Date** ________________

ROLLING A RECTANGULAR PRISM
(part 2) Converting Units

Metric
1 meter = 100 centimeters

Customary
1 yard = 3 feet =36 inches
1 foot = 12 inches

One student wrote the answers to the problems in cu. meters. What would his/her answers be in cu. centimeters?

1. Volume = 6 cu. meters Volume = _____cu. cm.
2. Volume = 3 cu. meters Volume = _____cu. cm.
3. Volume = 8 cu. meters Volume = _____cu. cm.
4. Volume = 11 cu. meters Volume = _____cu. cm.

One student wrote the answers to the problems in cu. feet. What would his/her answers be in cu. inches?

5. Volume = 4 cu. ft. Volume = _____cu. in.
6. Volume = 9 cu. ft. Volume =_____ cu. in.
7. Volume = 13 cu. ft. Volume = _____cu. in.
8. Volume = 7 cu. ft. Volume = _____cu. in.

One student wrote the answers to the problems in cu. inches. What would his/her answers be in cu. yards?

9. Volume = 36 cu. in. Volume = _____cu. yds.
10. Volume = 144 cu. in. Volume = _____cu. yds.
11. Volume = 72 cu. in. Volume= _____ cu. yds.
12. Volume = 108 cu. in. Volume= _____cu. yds.

CONSTRUCTING TASK:
Books, Books, and More Books!

In this task, students will determine the combined volume of three boxes of books. They will conclude that adding the volume of each box will give the combined volume.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MCC.5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MCC5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

SMP 1. Make sense of problems and persevere in solving them.
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SMP 4. Model with mathematics.
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SMP 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students will need to have had practice finding the volume of a rectangular prism. They will also need to recognize that addition can be used to combine rectangular prisms, just like they combine quantities by adding. Also, they will need to understand that real world problems require a variety of problem solving strategies.

ESSENTIAL QUESTIONS

- How can you find the combined volume of two or more rectangular prisms?
- How can you determine if your solution is correct?

MATERIALS

- Pencils
- Recording sheet

GROUPING

Individual/Partners

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will determine the combined volume of three boxes of books. They will conclude that adding the volume of each box will give the combined volume.

Comments: To introduce this task, tell them that you need to take three boxes of books home with you, but you are not sure they will fit in your truck. Tell them that they can help you figure out if they will fit, by figuring their volume. You may need to remind them of the formula for volume.

Task Directions: Determine the volume of each box of books and decide if they will all fit in the teacher's truck. Use pictures, words, and numbers to show your work.

FORMATIVE ASSESSMENT QUESTIONS

- What information do you need to be able to solve this problem?
- What is the largest size box you could fit, if all three boxes were the same size?

DIFFERENTIATION

Extension:

- Ask students if 4 boxes would fit?
- If your boxes were half the size of the originals, how many could you fit?

Intervention:

- Students may work with partners.
- Students may use calculators to determine volume.

TECHNOLOGY

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

Name ______________________________ **Date** ______________________________

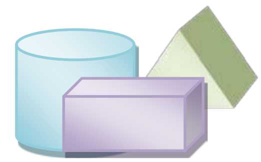
Books, Books, and More Books

Directions: Your teacher wants to take three boxes of books home from school. She needs to know if they will all fit in her truck, or if she needs to make two trips to get all the boxes home. Here is some information you will need:

- Two of the boxes are the same size. (2 ft. long, 3ft. wide, and 2 ft. high)
- One box is larger than the others. (3 ft. long, 3 ft. wide, and 3 ft. high)
- Your teacher's truck has 60 cu. ft of space.

Can your teacher take all three boxes in one load? Show how you know with pictures, words, and numbers.

PRACTICE TASK: Super Solids



In this task, students will estimate and find the volume of real-world objects.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

SMP 1. Make sense of problems and persevere in solving them.
SMP 2. Reason abstractly and quantitatively.
SMP 3. Construct viable arguments and critique the reasoning of others.
SMP 4. Model with mathematics.
SMP 5. Use appropriate tools strategically.
SMP 6. Attend to precision.
SMP 7. Look for and make use of structure.
SMP 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students should realize that square units represent 2-dimensional objects and have both length and width, while cubic units represent 3-dimensional objects and have length, width, and height.

Students should have had experiences with the attributes of rectangular prisms, such as faces, edges, and vertices, in fourth grade. This task will build upon this understanding.

The general formula for the area of a parallelogram is A = *bh*. Knowing the general formula for the area of a parallelogram enables students to memorize ONE formula for the area of rectangles, squares, and parallelograms since each of these shapes is a parallelogram.

The general formula for the volume of a prism is V = *Bh,* where *B* is the area of the BASE of the prism and *h* is the height of the prism. Knowing the general formula for the volume of a prism prevents students from having to memorize different formulas for each of the types of prisms they encounter.

COMMON MISCONCEPTIONS:

Students need to be encouraged to estimate the volume based on the information they have, but not actually calculating the answer. Estimating is not the same as guessing and students need to know that there are strategies involved in estimating. They need to be encouraged to share their strategies with each other.

ESSENTIAL QUESTIONS

- Explain the process of finding the volume of cubes and rectangular prisms?
- Why is volume represented with cubic units?
- What is the relationship between the volumes of geometric solids?
- How do we measure volume?

MATERIALS

- Empty boxes (such as shoe, cereal, cracker, etc.)
- Centimeter cubes
- Rulers or measuring tapes
- "Super Solids" task sheet

GROUPING

Partner/Small Group Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will estimate and find the volume of real-world objects.

Comments

For each object, students will estimate the number of centimeter cubes that will be needed completely fill the box. (They should NOT fill the box with centimeter cubes to estimate.) After all estimates have been recorded, students will use their measurement tools to determine the volume of each box. All measurements should be to the nearest tenth of a centimeter.

After students have found the volume of each box, compare results. Discuss any discrepancies. Allow pairs of students to share their strategies for making their estimate and determining the volume.

Task Directions

Students will follow the directions below from the "Super Solids" student recording sheet.

Objects to measure could include tissue box, storage tubs, lunch box, waste basket, storage area of desk, etc.

For each object you choose, estimate the number of centimeter cubes that will be needed to completely fill the box. Once you have recorded your estimate, measure the object to determine the volume of each box.

∗All measurements should be recorded to the nearest tenth of a centimeter.

Object	Estimate in cm^3	Area of Base $A = b \times h$	Height of Prism	Volume of Prism in cm^3 $A = B \times h$

FORMATIVE ASSESSMENT QUESTIONS

- What information did you use to help you estimate the volume of each rectangular prism?
- How did you find the area of the base of your prism?
- How did you find the volume of your prism?
- What is $\frac{1}{10} \times \frac{1}{10}$? What is 0.1×0.1? Where should you place your decimal in your answer? How do you know? (Students should recognize that $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$ and that $\frac{1}{100} \times \frac{1}{10} = \frac{1}{1{,}000}$. Therefore, $0.1 \times 0.1 = 0.01$ and $0.01 \times 0.1 = 0.001$.

DIFFERENTIATION

Extension

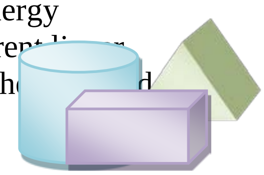
- Students can calculate the area of each surface of the solid and determine the total surface area.

Intervention

- Encourage students to fill their boxes with centimeter cubes. This allows students to use models when determining volume.

TECHNOLOGY

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but differen[illegible] dimensions. Students investigate relationships among the linear dimensions, th[illegible] d the volume of rectangular prisms.



Name____ Date______________________________

Super Solids

For each object you choose, estimate the number of centimeter cubes that will be needed to completely fill the box. Once you have recorded your estimate, measure the object to determine the volume of each box. ∗All measurements should be recorded to the nearest tenth of a centimeter.

Object	**Estimate in cm^3**	**Area of Base** $A = b \times h$	**Height of Prism**	**Volume of Prism in cm^3** $A = B \times h$

<u>Practice Task</u> - Survival Badge

In this activity, students will create line plots to evenly distribute a supply of water for a scout troop.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.

STANDARDS FOR MATHEMATICAL PRACTICE

SMP 1. Make sense of problems and persevere in solving
SMP 2. Reason abstractly and quantitatively.
SMP 4. Use appropriate tools strategically.
SMP 6. Attend to precision.
SMP 7. Look for and make use of structure.
SMP 8. Look for and express regularity in repeated reasoning.

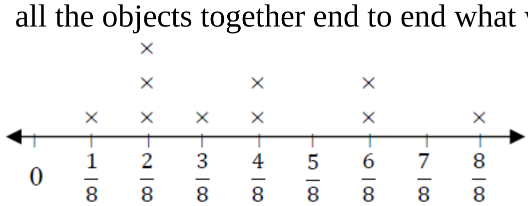
BACKGROUND KNOWLEDGE

One example of using line plots to solve real world problems might be illustrated in the following scenario. Given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were equal.

This standard provides a context for students to work with fractions by measuring objects to one-eighth of a unit. This includes length, mass, and liquid volume. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

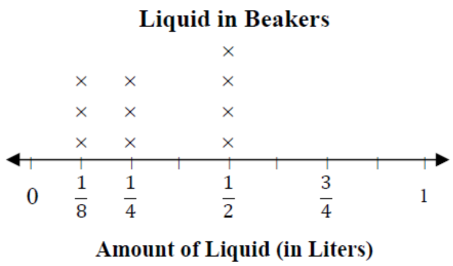
Example:

Students measured objects in their desk to the nearest $^1/_2$, $^1/_4$, or $^1/_8$ of an inch then displayed data collected on a line plot. How many objects measured $^1/_4$? $^1/_2$? If you put all the objects together end to end what would be the total length of **all** the objects?



Example:

Ten beakers, measured in liters, are filled with a liquid.



The line plot above shows the amount of liquid in liters in 10 beakers. If the liquid is redistributed equally, how much liquid would each beaker have? (This amount is the mean.)

Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers.

COMMON MISCONCEPTIONS

Students may not understand that in order to share the items equally, you must first find the total number of items. This portion of the standard gives them a visual model and becomes the background for finding the mean in grade 6.

ESSENTIAL QUESTIONS

- How can we use a line plot to show fractional parts of a whole?
- How can the information on the line plot be used to re-distribute the items equally?

MATERIALS

- Graph paper
- “Survival Badge” recording sheet

GROUPING

Pairs/small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

This task provides students with the opportunity to explore using information in a table to create a line plot. They will use the line plot to re-distribute the supply of water so that the same amount is in each canteen.

TASK

A Boy Scout Troop is working on a badge for survival. In order to earn the badge, they must decide how to use their available water supply equally. The water is in 12 canteens with varying amounts in each canteen. Students will use the data in the table to construct a line plot showing the various amounts of water in the canteens. Then they will re-distribute the water so that each canteen holds the same amount of water.

FORMATIVE ASSESSMENT QUESTIONS

- How can you show the various amounts of water in each canteen?
- How did you share the water equally?
- How do you know the amounts in the canteens are equal?

DIFFERENTIATION:

Extension

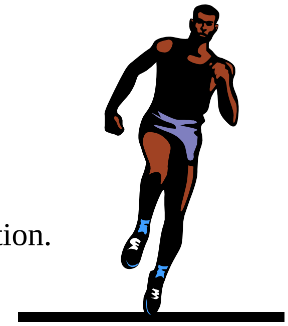
- Students can add additional canteens to their line plots and re-distribute the water again.

Intervention

- Students could use linking cubes to model the line plot and physically move them for re-distribution.

TECHNOLOGY

http://illuminations.nctm.org/LessonDetail.aspx?ID=L520 In this lesson, one of a multi-part unit from Illuminations, students conduct a survey based on a food court theme and then create pictographs and line plots.



Practice Task – A Little Mo Running

This task illustrates the distance Mo Farah ran when training for a marathon. Students will create a line plot and answer several questions based on the information.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.

STANDARDS FOR MATHEMATICAL PRACTICE

SMP 1. Make sense of problems and persevere in solving
SMP 2. Reason abstractly and quantitatively.
SMP 4. Use appropriate tools strategically.
SMP 6. Attend to precision.
SMP 7. Look for and make use of structure.
SMP 8. Look for and express regularity in repeated reasoning.

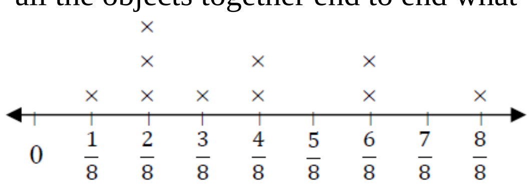
BACKGROUND KNOWLEDGE

One example of using line plots to solve real world problems might be illustrated in the following scenario. Given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were equal.

This standard provides a context for students to work with fractions by measuring objects to one-eighth of a unit. This includes length, mass, and liquid volume. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

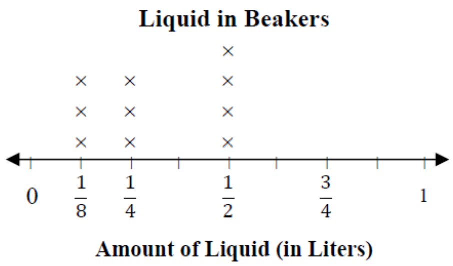
Example:

Students measured objects in their desk to the nearest $^1/_2$, $^1/_4$, or $^1/_8$ of an inch then displayed data collected on a line plot. How many objects measured $^1/_4$? $^1/_2$? If you put all the objects together end to end what would be the total length of **all** the objects?



Example:

Ten beakers, measured in liters, are filled with a liquid.



The line plot above shows the amount of liquid in liters in 10 beakers. If the liquid is redistributed equally, how much liquid would each beaker have? (This amount is the mean.)

Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers.

COMMON MISCONCEPTIONS

Students may not understand that in order to share the items equally, you must first find the total number of items. This portion of the standard gives them a visual model and becomes the background for finding the mean in grade 6.

ESSENTIAL QUESTIONS

- How can we use a line plot to show fractional parts of a whole?
- How can the information on the line plot be used to re-distribute the items equally?
- How can we manipulate the data on a line plot to answer questions?

MATERIALS

- Graph paper
- "A Little Mo Running" recording sheet

GROUPING

Individual/Pairs task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will analyze the data about Mo Farah's running routine to determine how many miles he should run after missing one day to maintain his total distance.

TASK

.Students will use a line plot to estimate the number of miles Mo needs to run. They will continue the task in part 2 where they will create a line plot about Mo's marathon completion and answer questions with the data.

FORMATIVE ASSESSMENT QUESTIONS

- How can using a line plot help you estimate the correct answer?
- How do you know your answer is correct?
- How can you illustrate fractional parts with a line plot?

DIFFERENTIATION:

Extension

- Students can research information about Mo Farah's races and create their own line plots using the data.

Intervention

- Students could use linking cubes to model the line plot and physically move them for re-distribution.

TECHNOLOGY

http://illuminations.nctm.org/LessonDetail.aspx?ID=L520 In this lesson, one of a multi-part unit from Illuminations, students conduct a survey based on a food court theme and then create pictographs and line plots.

PRACTICE TASK: Toy Box Designs

Adapted from K-5 Math Teaching Resources

In this task, students will be designing a toy box for a child's bedroom. The box needs to hold 30 cubic meters of toys. They must design two boxes with appropriate dimensions and tell which box would be most suitable for use in a child's bedroom.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MCC5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MCC5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

SMP 1. Make sense of problems and persevere in solving them.
SMP 2. Reason abstractly and quantitatively.
SMP 3. Construct viable arguments and critique the reasoning of others.
SMP 4. Model with mathematics.
SMP 5. Use appropriate tools strategically.
SMP 6. Attend to precision.
SMP 7. Look for and make use of structure.
SMP 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

They should also be familiar with using a metric ruler to measure and draw rectangular prisms. The knowledge of area and area of a base will be extended as students apply that understanding to volume of solids. Students should be familiar with using unit cubes to fill an object and find the volume. Students should also be able to use their knowledge of factors to determine the measurements for the box.

COMMON MISCONCEPTIONS:

Some students may think that the box must be a cube. They need to understand that rectangular prisms (boxes) can have different measures of length, width, and height. They will need to consider which design would work best for a child. For example, they could decide to use a height of 10 meters, width of 1 meter and length of 3 meters. However, a child could not practically use a toy box that is 10 meters tall.

ESSENTIAL QUESTIONS

- Explain the relationship between 3D objects that have different measurements, but the same volume.
- How did you determine the most appropriate dimensions for the box based on its use?

MATERIALS

- Ruler
- Paper (grid paper works very nicely)
- Centimeter cubes (optional)

GROUPING

Individual/pairs

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will be designing a toy box for a child's bedroom. The box needs to hold 30 cubic meters of toys. They must design two boxes with appropriate dimensions and tell which box would be most suitable for use in a child's bedroom.

Comments: You might begin this task by asking them if they have ever seen a toy box (a box designed to hold toys) and let them describe what they know. Ask them why they think the height of toy boxes is usually less than their width. Lead a general discussion of how the size of the toy box needs to be appropriate for use by a child.

Task Directions: Draw and label two designs for a toy box. Decide which design is most appropriate for a child's bedroom. Explain your answer.

FORMATIVE ASSESSMENT QUESTIONS

- How could you determine which 3 numbers could be multiplied together to get 30?
- Is your answer reasonable? How do you know?
- What expression might you use to find volume?

DIFFERENTIATION

Extension:

- Have students design another toy box with a capacity of 40 cubic feet.

Intervention:

- Students may work with partners.
- Students may use calculators.
- Students may use centimeter cubes to create a model.

TECHNOLOGY

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

Name_________________________ Date__________________________

Toy Box Designs

You are designing a toy box for child's bedroom. The toy box needs to be able to hold 30 cubic meters of toys. What might the dimensions be?

1. Draw and label two possible designs for the toy box.

2. Explain which design would work best in a child's bedroom and give reasons to support your choice.

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PRACTICE TASK: Breakfast for All

Adapted from K-5 Math Teaching Resources

In this task, students will be designing three different sizes of cereal boxes. They will need to determine the dimensions for the original box and then use the appropriate operations to enlarge or reduce the size of the original box to meet the specifications of the manufacturer.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MCC5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MCC5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

SMP 1. Make sense of problems and persevere in solving them.
SMP 2. Reason abstractly and quantitatively.
SMP 3. Construct viable arguments and critique the reasoning of others.
SMP 4. Model with mathematics.
SMP 5. Use appropriate tools strategically.
SMP 6. Attend to precision.
SMP 7. Look for and make use of structure.
SMP 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students should have had practice determining the volume of rectangular prisms. In addition, they should be familiar with the terminology "half the size of" and "three times the size of" and be able to determine relative dimensions. They should also be able to determine the correct unit of measure for given item (centimeters/inches or meters/feet/yards)

COMMON MISCONCEPTIONS:

Students may believe that in order to make the boxes "half the size" or "three times the size" they need to adjust each dimension (length, width, height) by half or three times. They need to investigate how the total volume is affected by changing the dimensions and determine "half" and "three time" by calculating total volume.

ESSENTIAL QUESTIONS

- How can I determine appropriate units of measure for an object?
- How did you determine the sizes for the mini-sized box and the super-sized box?

MATERIALS

- Ruler
- Grid paper

GROUPING

Individual/Pairs

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will be designing three different sizes of cereal boxes. They will need to determine the dimensions for the original box and then use the appropriate operations to enlarge or reduce the size of the original box to meet the specifications of the manufacturer.

Comments: You could begin this task by showing several cereal boxes and asking them to estimate the dimensions of the box. They could even measure a cereal box to find out what the appropriate dimensions could be.

Task Directions: Design the packaging for a new breakfast cereal in three different sized boxes. Draw a design for each box. Label the dimensions and calculate the volume of each one.

FORMATIVE ASSESSMENT QUESTIONS

- Justify why you chose which unit of measure to use.
- Is your answer reasonable? How do you know?

DIFFERENTIATION

Extension:

- Have students find the answer to the following question: *How big would a box have to be to hold enough cereal for your entire school?*

Intervention:

- Students may work with partners.
- Students may use calculators.

TECHNOLOGY

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

Name ____ Date ________________________

Breakfast for All

You have been asked to create the packaging for a new kind of cereal. The manufacturer wants three different sized boxes:

1. A standard sized cereal box
2. A mini sized box that is half as tall, half as wide, and half as deep as the standard size
3. A super sized box that is three times as tall, three times as wide and three times as deep as the standard size.

Using grid paper, draw a possible design for each box. Label the dimensions and calculate the volume.

Which box do you think would be the best seller? Write your answer on the lines below and tell why you think so.

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Culminating Task: Boxing Boxes

In this task, students explore volume while packing shipping boxes with various-sized merchandise boxes.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MCC5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MCC5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

MCC5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

SMP 1. Make sense of problems and persevere in solving them.
SMP 2. Reason abstractly and quantitatively.
SMP 3. Construct viable arguments and critique the reasoning of others.
SMP 4. Model with mathematics.
SMP 5. Use appropriate tools strategically.
SMP 6. Attend to precision.
SMP 7. Look for and make use of structure.
SMP 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

"*Volume* typically refers to the amount of space that an object takes up" whereas "*capacity* is generally used to refer to the amount that container will hold," Van de Walle (2006) (p. 265). To distinguish further between the two terms, consider how the two are typically measured. Volume is measured using linear measures (ft, cm, in, m, etc) while capacity is measured using liquid measures (L, mL, qt, pt, g, etc). However, Van de Walle reminds educators, "having made these distinctions [between volume and capacity], they are not ones to worry about. The term *volume* can also be used to refer to the *capacity* of a container" (p. 266).

Van de Walle, J. A. & Lovin, L. H. (2006). *Teaching students-centered mathematics: Grades 3-5*. Boston: Pearson Education, Inc.

COMMON MISCONCEPTIONS

When solving problems that require renaming units, students use their knowledge of renaming the numbers as with whole numbers. Students need to pay attention to the unit of measurement which dictates the renaming and the number to use.

ESSENTIAL QUESTIONS

- Can different size containers have the same volume?
- Why does the formula *V=l x w x h* work to find volume?

MATERIALS

- "Boxing Boxes" student recording sheet
- Snap cubes and/or 1" grid paper (several sheets per student), scissors, and clear tape
- "Boxing Boxes, Part II" student recording sheet (optional)

GROUPING

Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students explore volume while packing shipping boxes with various-sized merchandise boxes.

Comments

This task can be introduced by asking small groups of students to create the different sized merchandise boxes using grid paper or snap cubes. If using grid paper, students will need to sketch the nets for the boxes described on 1" grid paper and then cut the nets out and fold them to create the rectangular prisms. If using snap cubes, students can create the required rectangular prisms with snap cubes using the dimensions required. Students can then use these models while working on the task.

Allow students to create their own chart for the "Boxing Boxes" task that makes sense to them. Then allow students to share their chart with students in their small group and choose two or three students who created different charts to share their work with the class.

Notice that the capacity of the standard shipping box is 12 ft^3. Therefore, the sum of the volumes of the merchandise boxes packed must equal 12 ft^3 for each packing plan (see table below).

Volume of Merchandise Boxes (Number of Boxes × Volume of each Box in ft^3)					
Packing Plans	**Merchandise Box W**	**Merchandise Box X**	**Merchandise Box Y**	**Merchandise Box Z**	**Total Volume**
1				**12 × 1 ft^3**	**12 ft^3**
2	**2 × 6 ft^3**				**12 ft^3**
3	**1 × 6 ft^3**	**1 × 4 ft^3**		**2 × 1 ft^3**	**12 ft^3**
4	**1 × 6 ft^3**			**6 × 1 ft^3**	**12 ft^3**
5		**3 × 4 ft^3**			**12 ft^3**
6		**2 × 4 ft^3**		**4 × 1 ft^3**	**12 ft^3**
7			**1 × 8 ft^3**	**4 × 1 ft^3**	**12 ft^3**
8		**1 × 4 ft^3**		**8 × 1 ft^3**	**12 ft^3**

Merchandise Packing Guide

Number of Merchandise Boxes in Each Standard Packing Box				
Packing Plans	**Merchandise Box W**	**Merchandise Box X**	**Merchandise Box Y**	**Merchandise Box Z**
1				12
2	2			
3	1	1		2
4	1			6
5		3		
6		2		4
7			1	4
8		1		1

The volume of the merchandise boxes are as follows:
Merchandise Box W: 1 ft x 3 ft x 2 ft = 6 ft^3
Merchandise Box X: 1 ft x 2 ft x 2 ft = 4 ft^3
Merchandise Box Y: 2 ft x 2 ft x 2 ft = 8 ft^3
Merchandise Box Z: 1 ft x 1 ft x 1 ft = 1 ft^3

The capacity of the standard shipping box is 2 ft × 3 ft × 2 ft = 12 ft^3

Additionally, students will need to write a letter to their boss explaining how to use the chart they created.

Task Directions

Students will follow the directions below from the "Boxing Boxes" student recording sheet.

You have been hired by Boxes Unlimited to help determine the best way to package merchandise for shipping.

Boxes Unlimited has a standard shipping box which will hold merchandise measuring 2 ft by 3 ft by 2 ft.

Boxes Unlimited needs to pack merchandise they receive into the standard shipping box. The merchandise arrives in four different box sizes.

Merchandise Box W is 1 ft. x 3 ft. x 2 ft.
Merchandise Box X is 1 ft. x 2 ft. x 2 ft.
Merchandise Box Y is 2 ft. x 2 ft. x 2 ft.
Merchandise Box Z is 1 ft. x 1 ft. x 1 ft.

Your task is to create a chart for employees to use as a reference when they are packing boxes for shipment. Be sure to include the volume of each merchandise box and the capacity of the standard shipping box on your chart. Convert the capacity of the standard shipping box from cubic feet to cubic yards.

Write a report to your boss explaining how to read your chart.

FORMATIVE ASSESSMENT QUESTIONS

- Have you found all of the possible ways to fill the standard shipping box? How do you know?
- What is the total capacity of the standard shipping box? Will the merchandise completely fill the standard shipping box? How do you know?
- How are you organizing your packing chart? Why did you choose this type of organizational chart?
- Explain how your chart could be used by the employees who pack boxes?
- Will the formula *l x w x h* work to find the volume of any 3D shape? Explain your reasoning.

DIFFERENTIATION

Extension

- Ask students to consider a large shipping box with dimensions of 3 ft × 3 ft × 3 ft. What are the ways that this packing box could be filled with the given merchandise boxes? Students could work the task with this large shipping box rather than the regular shipping box. Next, students who worked with the large shipping box could be paired with students who worked on the standard shipping box. Partners could then be asked to determine which size box would be a better choice and justify their thinking.

Intervention

- Encourage students to use snap-cubes to create models of the merchandise boxes.
- Students who would benefit from a chart in which to record their work should be provided one. A sample is given below. See "Boxing Boxes, Part II" student recording sheet.

TECHNOLOGY

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear

dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

Name____ Date ______________________________

Boxing Boxes

You have been hired by Boxes Unlimited to help determine the best way to package merchandise for shipping.

Boxes Unlimited has a standard shipping box which will hold merchandise measuring 2 ft by 3 ft by 2 ft.

Boxes Unlimited needs to pack merchandise they receive into the standard shipping box. The merchandise arrives in four different box sizes.
Merchandise Box W is 1 ft. x 3 ft. x 2 ft.
Merchandise Box X is 1 ft. x 2 ft. x 2 ft.
Merchandise Box Y is 2 ft. x 2 ft. x 2 ft.
Merchandise Box Z is 1 ft. x 1 ft. x 1 ft.

Your task is to create a chart for employees to use as a reference when they are packing boxes for shipment. Be sure to include the volume of each merchandise box and the capacity of the standard shipping box on your chart. Convert the capacity of a standard shipping box from cubic feet to cubic yards.

Write a report to your boss explaining how to read your chart.

Name____ Date ________________________________

Boxing Boxes
Part II

The volume of the merchandise boxes are as follows:

Merchandise Box W: __________________________________

Merchandise Box X: __________________________________

Merchandise Box Y: __________________________________

Merchandise Box Z: __________________________________

The capacity of the standard shipping box is __________________________________.